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Compensation of Integrator Time Constants for Electric Field Measurements

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***Abstract*—Rubinstein et al.** [1] **presented the measurement of the electric field strength during lightning discharge with analogue integrators as amplifiers and their numerical correction of the time constant that is needed by means of stability of the integrator. We have extended his work by incorporating the antenna characteristic into the system equations. In this paper we focus on the compensation of the integrator time constant (Eq. (13),** [1]**). We also defined the parameter k_a which was introduced in Eq. (6) of** [1]**. Further, we analyzed and present results of that compensation method for time synchronized E-field measurements with two different integrators, so called E-slow and E-fast, recorded in Sao Paulo City on March 1st, 2014. In this context we discovered the importance of offset errors that exist. It will be discussed, why the offset of the system has a large influence while using the method of compensation and a simple approach for handling the offset will be presented. Additionally we show that this compensation method can be used to determine continuing currents by applying this method to fast E-fields. To verify this we used a sample of recorded current and fast E-field at Gaisberg Tower in Salzburg, Austria. The advantage of the compensation method for E-fast in comparison to an almost ideal integrator (E-slow) regarding the gain and quantization noise will be mentioned in the conclusion. The Appendix A contains the description of the system in the Laplace domain with useful simplifications, and shows the Bode plots.**

Keywords—lightning, E-fast, E-slow, time constant, compensation, convolution, offset error, continuing current, quantisation noise

I. INTRODUCTION

Lightning electromagnetic fields are often measured with so called flat plate antennas. The output of the flat plate antennas is integrated either with an active or passive circuit because the antenna itself measures the derivative of the electromagnetic field. Depending on the RC time constant of the integrator the resulting fields are called slow E-field or fast E-field.

Recently Rubinstein et al. [1] presented a method to transfer electromagnetic fields measured with a RC time constant τ_1 to a theoretical time constant τ_2. This gives the possibility to compare electromagnetic fields measured with different integrators (different time constants). In the paper of Rubinstein et al. [1] the transfer only applies to different integrator time constants. In this paper we will extend the method described in [1] by the characteristics of the antenna. The characteristics of the antenna mainly influence the gain of the system. If two systems with different characteristics (antenna diameters and/or time constants) are considered, there are two possible ways to approach a comparison of the two systems.

One possibility is to transfer the Laplace-domain system transfer function from the system with the faster time constant to the one with slower time constant (as it is done in Rubinstein et al. [1]). The waveform resulting from the convolution of the time-domain representation of this transfer function with the measured E-field of the fast system then should look similar to the waveform measured with the slow system. That makes the systems comparable. We call this method “E-field Transfer”.

The other method is compensation, where for the system transfer function H(s) with $V_o(s) = E(s)\,H(s)$ the time-domain representation of $\frac{1}{H(s)}$ has to be determined. E(s) is the electric field strength which shall be measured with the antenna and $V_o(s)$ is the output voltage of the integrator. In that case the convolution yields the unbiased E-field. The resulting waveforms of both systems should be identical. We call this method “E-field Compensation”.

Lightning electromagnetic fields compensated for time constant are often used to determine the continuing current of a return stroke [2], [3], [4], [5]. Normally so called “slow” antennas (antennas with time constant greater than 1 s) are used. The resulting field is then “compensated” for the time constant by a “graphical” method [6], [7].

It is the goal of this paper to show that so called fast E-field data (measured with integrators with a small time constant, e.g. 0.5 ms) can also be used to determine continuing currents as with slow E-fields. The Compensation method described here is maybe even more accurate than “graphical convolution” of E-field data measured with “slow” antennas.

In this paper we will first describe the different systems that are used to measure the electric field of the lightning discharge, show some results, and finally give a conclusion.

Especially the compensation method considering the antenna characteristics and offset error treatment shall be presented.

II. Methods of Measurement

As already mentioned in the introduction it is possible to integrate the measured dE/dt of the antenna output with an active or passive circuit.

The main characteristics are an amplifier-specific gain k and a time constant τ which makes the integrator stable. The following two graphics show a schematic of the two measurement arrangements:

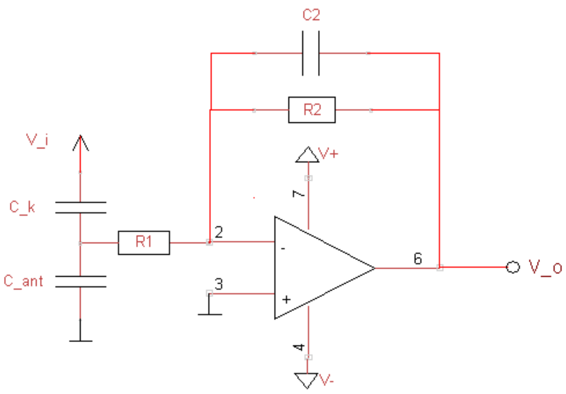


Fig. 1a: Antenna with integrator (active curcuit)

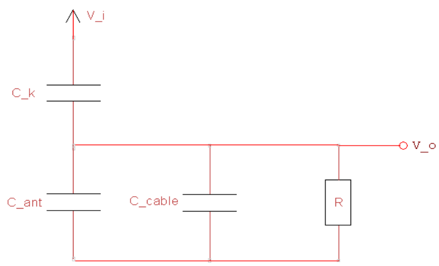


Fig. 1b: Antenna without integrator (passive circuit)

In Fig.1a and Fig.1b C_k represents the capacity of a calibration plate that is used for testing only. It is not used for the measurement of E-fields and therefore C_k is zero but it is necessary for the analytical calculation of the transfer function. C_ant is the capacity of the antenna. C_cable is the cable capacity which is more important for the system without integrator.

The corresponding Laplace domain transfer function for the active integrator is:

$$H_{int}(s) = \frac{V_{out(s)}}{E(s)} = s\,\frac{d_{ant}}{\frac{1}{\tau_{ant}}+s}\,\frac{-k_{int}}{\frac{1}{\tau_{int}}+s} \qquad (1a)$$

where $k_{int} = \frac{1}{R_1 C_2}$, $\tau_{ant} = R_1 C_{ant}$ and $\tau_{int} = R_2 C_2$

The indices 'int' and 'ant' stand for 'integrator' and 'antenna'. For the passive integrator the corresponding transfer function is:

$$H_{ni}(s) = d_{ant}\,\frac{C_{ant}}{C_{ant}+C_{cable}}\,\frac{s}{\frac{1}{R\,(C_{ant}+C_{cable})}+s} \qquad (1b)$$

where $\frac{C_{ant}}{C_{ant}+C_{cable}} = k_{ni}$ and $R\,(C_{ant}+C_{cable}) = \tau_{ni}$

The index 'ni' stands for 'no integrator'.

In order to compensate the time constant and gain of the measured signal to get the physical E-field, V_{out} must be multiplied with $H_{int}^{-1}(s)$ or $H_{ni}^{-1}(s)$, respectively. After simplification (see Appendix A) these can be written as:

$$H_{e_{int}}(s) = H_{int}^{-1}(s) = \frac{-1}{k_{damp}\,k_{int}}\left(\frac{1}{\tau_{int}}\frac{1}{s}+1\right) \qquad (2a)$$

$$H_{e_{ni}}(s) = H_{ni}^{-1}(s) = \frac{1}{d_{ant}\,k_{ni}}\left(\frac{1}{\tau_{ni}}\frac{1}{s}+1\right) \qquad (2b)$$

The time domain functions for compensation (inverse Laplace transform of the Eq. (1a) $H_{e_{int}}$ and Eq. (1b) $H_{e_{ni}}$) which are convolved with the measured field are given in Eq. (3a) and Eq. (3b).

$$h_{e_{int}}(t) = \frac{-1}{k_{damp}\,k_{int}}\left(\frac{1}{\tau_{int}}\sigma(t)+\delta(t)\right) \qquad (3a)$$

or simplified: $h_{e_{int}}(t) = -\frac{1}{\varepsilon_0\,A\,R_2}\sigma(t) - \frac{C_2}{\varepsilon_0\,A}\,\delta(t)$

$$h_{e_{ni}}(t) = \frac{1}{d_{ant}\,k_{ni}}\left(\frac{1}{\tau_{ni}}\sigma(t)+\delta(t)\right) \qquad (3b)$$

or simplified: $h_{e_{ni}}(t) = \frac{1}{\varepsilon_0\,A\,R}\sigma(t) + \left(\frac{1}{d_{ant}}+\frac{C_{cable}}{\varepsilon_0\,A}\right)\delta(t)$,

where σ(t) is the unit step, δ(t) is the dirac pulse, A is the surface area of the antenna, d_{ant} the distance of the antenna plate from the other electrode (parallel plate capacitor model) and ε_0 is the vacuum permittivity.

These functions are convolved with the measured E-fields. The convolution with the δ(t) part of the compensation function scales the high frequencies, which are well integrated by the system (see Bode plot in Appendix). The convolution with the σ(t) part corrects the time constant of the integrator, which is used for the measurements. For $R_2 \rightarrow \infty$ one would have an ideal integrator and the compensation degenerates to a scaling (gain correction or calibration factor correction) of the waveform only. In section III we will discuss the influence of the gain correction (δ(t) part) for the high frequency content in the waveform of either the slow or the fast time constant integrator system.

For the simplified form of Eq. (3a) and Eq. (3b) for example, it can be seen which influence the parameters have on the convolution result. Especially for the system with passive integrator (see [1], Fig. 3) the compensation function is similar in form but in contrast, the cable capacity is not a negligible parameter. It is crucial to know the cable capacity exactly, if that method of measurement is used. $\frac{1}{d_{ant}}$ is small compared to $\frac{C_{cable}}{\varepsilon_0\, A}$ and therefore negligible.

Further, the simplified Eq. (3a) can be convolved with a measured signal $V_o(t)$ resulting in

$$V_o^{New}(t)\;[=E(t)] = -\frac{C_2}{\varepsilon_0 A}V_o(t) - \frac{1}{\varepsilon_0\, A\, R_2}\int_0^t V_o(\kappa)d\kappa \qquad (4)$$

Eq. (3a) and Eq. (4) can be compared to Eq. (13) from [1]. The calibration factor k_a from [1] can be found in Eq. (4) $k_a = -\frac{1}{k_{damp}k_{int}} = -\frac{C_2}{\varepsilon_0 A}$. Calculation of $\frac{k_a}{\tau_{int}}$ gives the second factor of Eq. (4).

A potential problem of this method is any offset of the measured waveform. Even a minimal offset leads to noticeable deviations of the expected results, as the convolution sums up the offset values for the convolution interval. A method to circumvent this problem will be presented in section III.

III. DATA ANALYSIS (EXAMPLES)

The measurement of the data used in the following section was done in Sao Paulo City on March 1st, 2014. The lightning strike distance was 8 km. The equipment used is the same as described in [5].

The time constant of E-slow is 1700 ms and E-fast 0.47 ms. The measurement was done with a 5 MS/s digitizer where 1s files are recorded continuously. The data are processed in Scilab (www.scilab.org) and the program can compensate data samples of up to 1s.

Figs. 2 and Fig. 3 show the application of the above discussed derivations for the system with integrator. The measured E-fields of the 'slow' system and the 'fast' system and their corresponding spectrum are shown in Fig. 2a and Fig. 2b, respectively. The blue curve in Fig. 2b shows the spectrum of E-fast. The system with the slow time constant can measure the E-field down to quite low frequencies in comparison to the system with a faster time constant. This is obvious when the two spectra are compared. In Fig. 2b for E-slow (black curve) the gain for low frequencies is much higher than for E-fast (blue curve). Additionally Fig. 2b also shows the difference of the calibration factor (antenna gain). A multiplication with a constant means, that a constant is added in the logarithmic plot. This is the nearly constant vertical shift of the spectra for frequencies greater than about 500 Hz, where both systems should already work as 'ideal' integrators.

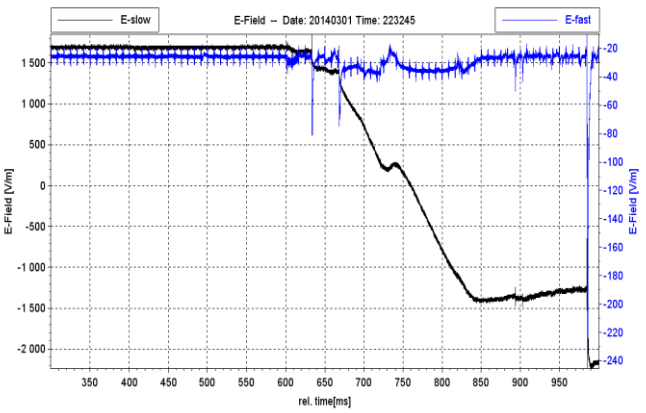


Fig. 2a: Uncompensated E-fields with slow (black) and fast (blue) integrator with respect to the axis scaling

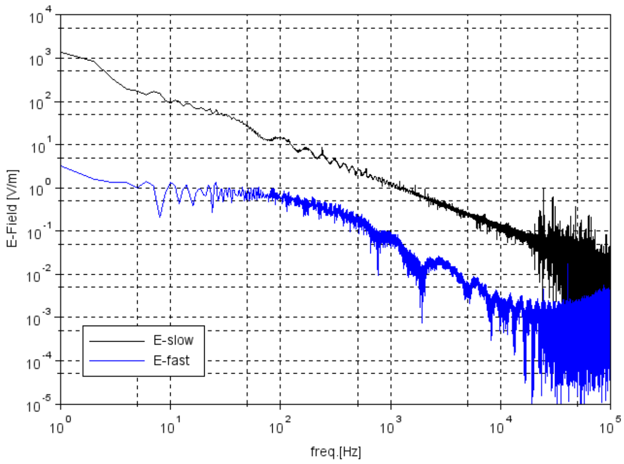


Fig. 2b: Spectrum of the uncompensated channels (E-slow: black, E-fast: blue)

Further, in Fig. 3a the method of compensation has been applied to the E-fields of Fig. 2a from 600 ms to 1000 ms with an offset error correction (which is yet to be discussed). The result is a good agreement of the two waveforms, as it would be expected after compensation with Eq. (3a). The waveforms can now be interpreted as the actual physical development of the electric field strength, at the measurement site. Fig. 3b shows the result of the spectra after compensation. They are almost identical. The reason that the noise of the black curve (in this case from E-slow) still exists is the longer time constant of the integrator which causes a worse 'noise filter' than for a short time constant. This can be seen in Eq. (3a)). In the convolution, the $\sigma(t)$-part of the equation is responsible for integration which suppresses high frequency parts, such as the (quantization) noise (Fig. 2b) above 10 kHz). Convolution with the $\delta(t)$-part on the other hand gives a scaled copy of the measured waveform as a result. These two are added to get the compensated waveform. Now, if τ_{int} is small, then the $\sigma(t)$-part will dominate in comparison to the $\delta(t)$-part and the noise from the measured E-field will have only small weight in the result of the compensation. If τ_{int} is high, the $\delta(t)$-part and accordingly the measured field will dominate in the result, hence the noise in the high frequency range is hardly reduced. That behavior can be observed in the comparison of the spectra in Fig. 3b, where both plots represent the compensated waveforms. For the time domain, Fig. 6 gives a good example of what a high quantization noise of a low gain, slow time constant system, looks like in comparison to a compensated, fast time constant system. This means that from noisy recordings of E-fields with the fast antenna useful signals can be recovered very well.

The E-slow does generally not change its shape too noticeably, as it is already close to an ideal integrator.

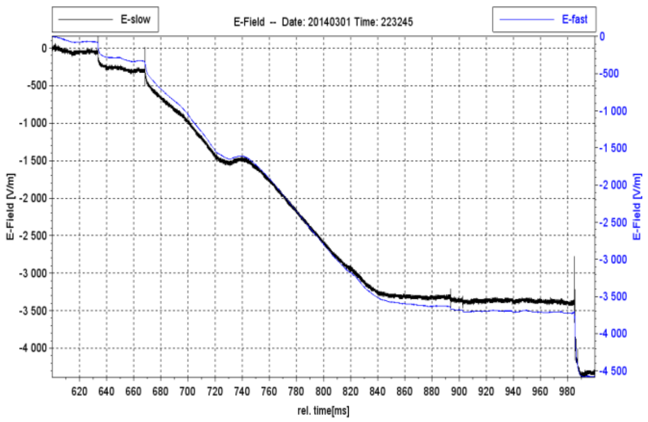


Fig. 3a: Compensated E-fields channels (E-slow: black, E-fast: blue

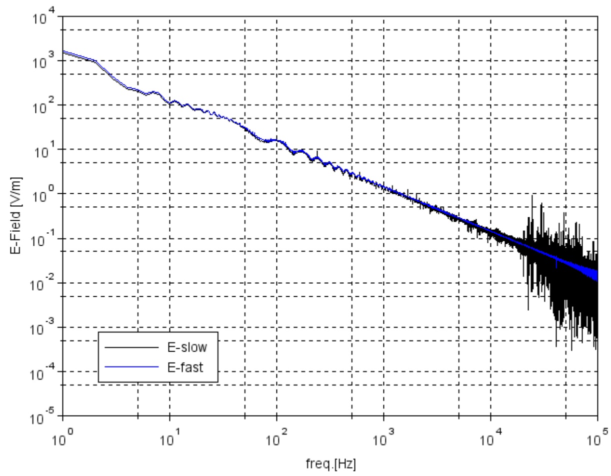


Fig. 3b: Spectrum of the compensated E-fields (E-slow: black, E-fast: blue)

One problem of the compensation with the convolution operation is that an offset of the system (which is due to offset voltages/currents of the OP-amplifier and may be assumed to be constant) causes a deviation of the expected result. Fig. 4 demonstrates what happens, if the offset error is not considered. The green curve is the compensated E-fast. It drifts away, as the convolution sums up all the offsets over the 1s recording. If on the other hand, the convolution is done for a small period only (for example for the duration of several hundred µs up to 1ms), this effect will be hardly observable if the offset error is small. This is the case in [1] where the convolution was performed on a 120 µs sample.

Any existing offset can be determined in time periods of the current sample or previous samples, where no lightning activity is observed and therefore the graphs seem to have no rates of change. The mean value over a period of time will give an approximation of the offset error. This value is subtracted from the whole sample before compensation. Fig. 4 shows the compensated E-fast from Fig. 2a with and without offset error correction. In most cases, the offset error is almost constant over 1s.

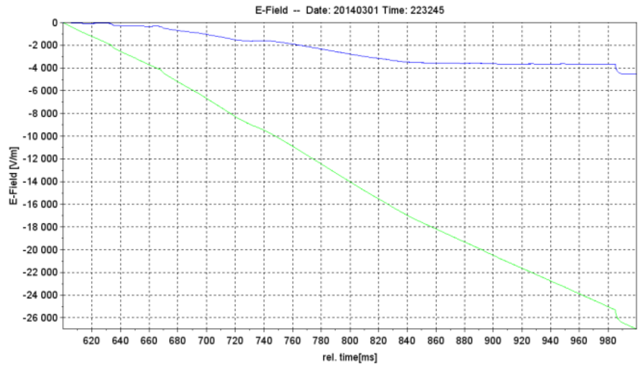


Fig. 4: Compensated E-fast shown in Fig. 2a without offset error correction (green) and compensated E-fast with offset correction (blue)

As another test for the presented algorithm, we used three directly measured strokes with continuing current at the Gaisberg Tower in Austria and their corresponding time-correlated E-fast measurements to reproduce the continuing current with the compensated fast E-fields using the method described in [2], [3], [4], [5]. The methods to calculate the continuing current require an assumption about the point charge height which was in our case assumed to be 7 km. Fig. 5 shows as example the measured and calculated continuing current for a return stroke of flash #561.

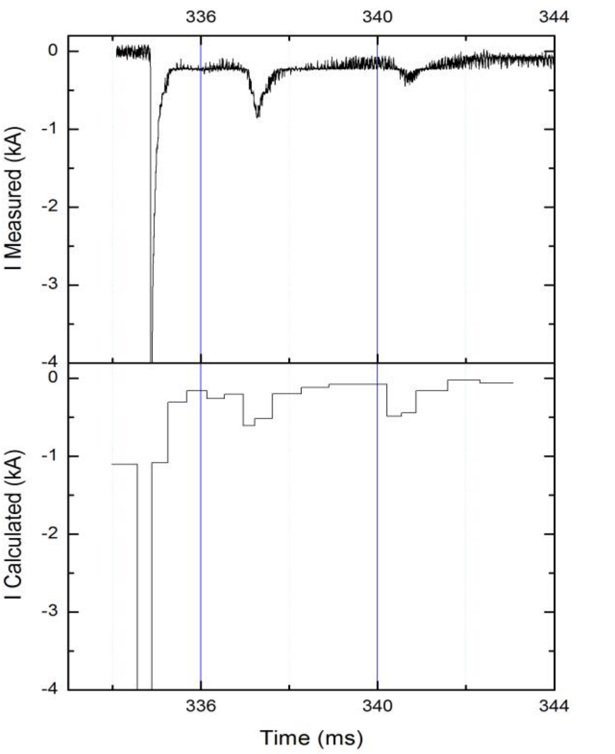


Fig. 5: Measured and calculated continuing current for a return stroke of flash #561 to the Gaisberg tower.

For all the three continuing currents the shape of the continuing current could be reproduced nicely. Nevertheless the assumed height of the charge with 7 km is in our experience too high. For realistic charge heights of about 4 to 5 km the resulting current is too low. This difference could be related to the measurement very close to the Gaisberg tower (172 m) because in such close ranges the assumptions of the model to calculate the continuing currents from E-fields are maybe no longer valid.

IV. Conclusion

This work extends the concept of [1] and focusses on the method of compensation. It can be seen that the performance is very good even for low SNR. Presented compensation examples show, that a fast system can be compensated for its time constant with a result that is very good compared to a system that integrates closer to an ideal integrator. Once the correct parameters (e.g. time constants, gain, size of the antenna) of the system with a flat plate antenna and integrator are found, the compensation function can be adapted for any system similar to the one presented in that paper. Further we determined the constant k_a of Eq. (13) in Rubinstein et al. [1] qualitatively. The offset error treatment is a crucial topic when performing the compensation over a long time or when the offset appears to be large. We presented a simple approach, how to determine this offset that the integrator produces due to the operational amplifier offset currents and voltage. A stable offset behavior of the system with slow rates of change is eligible. If there is a larger set of consecutive measurements available, it is advisable to check the seconds before the time of interest and to analyze the offset behavior. In many cases it is even possible to determine the offset error in one of the previous samples before lightning activity, where the signals look similar to Fig. 2a from 300 ms to 600 ms and then apply it on the sample of interest.

As a further conclusion, it can be considered to measure and process the lightning E-fields only using integrators with fast time constants because of a better signal resolution that this setup provides. The signal resolution of the compensation result will therefore be good as well. For example, due to its coarse signal resolution, the slow system is not able to reproduce the radiation field peak of return strokes. An E-field compensated with the presented methods on the other hand, is able to retain that part of the signal (see Fig. 6). This means, that the waveform of a compensated E-fast system would (ideally) contain information about return strokes with its fast varying parts, such as the radiation field peak, continuing currents, and slow variations of cloud charge all together. The numerical calculation tools as Matlab® or Scilab and others have a high performance on common computers that are used nowadays. Hence, the convolution can be done for many data files in reasonable time in order to gain further information for lightning research.

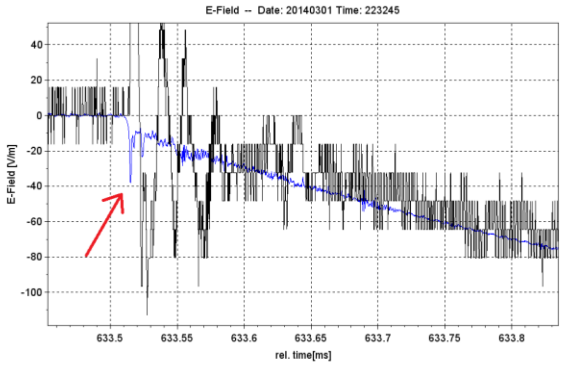


Fig. 6: Black line: compensated E-slow. Blue line: compensated E-fast. The radiation peak is highlighted with the red arrow

V. Appendix A: System Description

The Laplace-domain transfer function of the system with integrator given by Eq. (1a) can be divided into a differentiating part D(s):

$$D(s) = s,$$

an Antenna Damping K(s):

$$K(s) = \frac{d_{ant}}{\frac{1}{R_1 C_{ant}} + s},$$

and Integrator I(s):

$$I(s) = \frac{-\frac{1}{R_1 C_2}}{\frac{1}{R_2 C_2} + s}.$$

The goal of that separation is the following: Rubinstein et al. [1] derived the transfer method from the Laplace-domain transfer function of the integrator $\frac{V_o(s)}{Vi(s)} = \frac{-k}{s+\frac{1}{\tau_f}}$ with $k = \frac{1}{R_1 C_2}$ and $\tau_f = R_2 C_2$. It has exactly the same form as the term 'Integrator I(s)' and therefore makes H_{int} (Eq. (1a)) more descriptive in comparison to [1]. The two terms Differentiator D(s) (ideal differentiator which has a high pass characteristic: not feasible but useful for theoretical interpretation) and Antenna Damping K(s) (low pass filter with a very high border frequency) together form a band pass filter (see Bode plot in Fig. 7a) and represent the differentiating and damping behavior of the antenna with respect to geometrical factors of the plate antenna. The sense of the separation of D(s) and K(s) is on the one hand, that the K(s) simplifies to a damping constant k_{damp} (easily evaluated at 0 Hz giving $k_{damp} = R_1 \varepsilon_0 A$) in the relevant frequency range (below sampling rate) and, on the other hand, when $H_{int}^{-1}(s) = \frac{1}{k_{damp} D(s)\, I(s)}$ is calculated, $\frac{1}{D(s)}$ cancels the s from the numerator of $\frac{1}{I(s)}$. Then it is possible to get a time-domain function out of the inverse Laplace transformation without the need of time-discrete derivation, which makes the implementation easier.
The Bode plot of the complete Laplace-domain transfer function $V_{out}(s) = H_{int}(s)\, E(s)$ is depicted in Fig. 7b. The red vertical line marks the frequency limit of measurement given by the sampling rate of the digitizer. In our case, the upper frequency bandwidth is determined by the sample rate of the system (5 MHz, vertical red line in Fig. 7). The most important part for the compensation is the frequency range from 0-1 kHz.

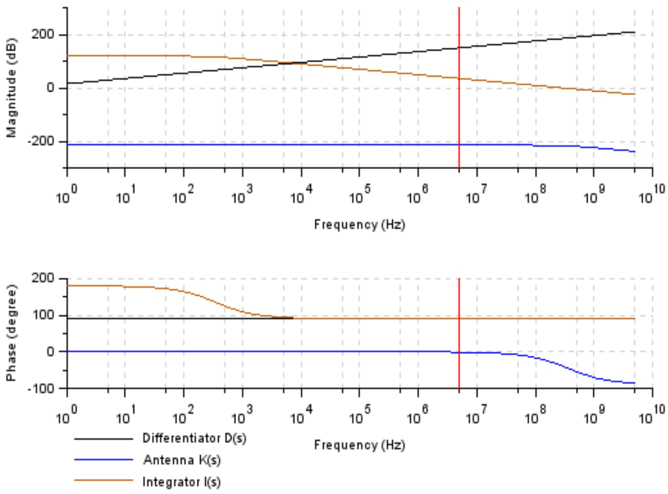


Fig. 7a) Bode plot of individual transfer functions
(red vertical line: sample rate)

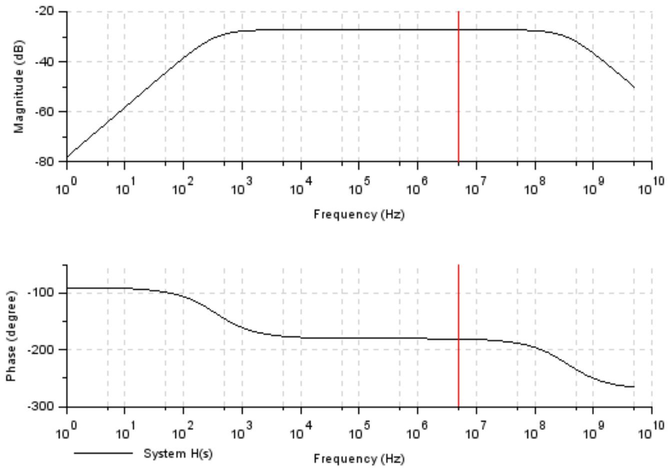
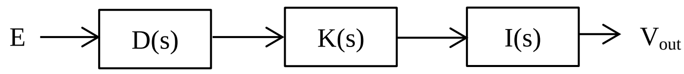


Fig. 7b) Bode plot of the complete system
(red vertical line: sample rate)



This block diagram is for interpretation only, to get insight that the system differentiates with respect to time (t), damps and integrates. The blocks are NOT free of feedback. Same parameters (resistors and capacities) can occur in more than one of the blocks.

Parameters used for the bode plot shown in Fig. 7 are $R_1 = 51\ \Omega$, $R_2 = 45.7\ M\Omega$ and $C_2 = 10$ pF and antenna surface diameter d = 0.25 m.

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REFERENCES

[1] M. Rubinstein, J.-L. Bermúdez, V. A. Rakov, F. Rachidi, and A. M. Hussein, "Compensation of the Instrumental Decay in Measured Lightning Electric Field Waveforms," *Electromagn. Compat. IEEE Trans.*, vol. 54, no. 3, pp. 685–688, 2012.

[2] M. Brook, N. Kitagawa, and E. J. Workman, "Quantitative study of strokes and continuing currents in lightning discharges to ground," *J. Geophys. Res.*, vol. 67, no. 2, pp. 649–659, 1962.

[3] T. Shindo and M. A. Uman, “Continuing current in negative cloud-to-ground lightning,” *J. Geophys. Res. Atmos.*, vol. 94, no. D4, pp. 5189–5198, 1989.

[4] M. A. Ross, S. A. Cummer, T. K. Nielsen, and Y. Zhang, “Simultaneous remote electric and magnetic field measurements of lightning continuing currents,” *J. Geophys. Res. Atmos.*, vol. 113, no. D20, p. D20125, 2008.

[5] C. Schumann and M. M. F. Saba, “Continuing current intensity in positive ground flashes,” in *Lightning Protection (ICLP), 2012 International Conference on*, 2012, pp. 1–5.

[6] V. Mazur and L. H. Ruhnke, “Determining the striking distance of lightning through its relationship to leader potential,” *J. Geophys. Res. Atmos.*, vol. 108, no. D14, p. 4409, 2003.

[7] E. de C. Ferraz, “Measuring of Continuing Currents in Natural Negative Cloud-to-Ground Lightning on Brasil: Development of Equipment and First Results,” *PhD Thesis*, *INPE*, 2009.