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# On the Effect of the Finite Ground Conductivity on Electromagnetic Field Radiated by Lightning to Tall Towers

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**Abstract--** In this paper it is shown how the finitely conducting ground modifies the signature of the radiation field of return strokes striking tall towers. Results are presented for different tower heights and for different ground conductivities varying the current risetime in the return stroke model. The results show that the attenuation of the initial peak of the radiation field resulting from the propagation over finitely conducting ground depends strongly on the current risetime, the tower height and the ground conductivity. In general, the attenuation of the radiation field of lightning flashes striking tall towers is larger than that striking flat ground. In the case where the ground conductivity is extremely poor, namely 0.0001 S/m, the attenuation of the peak radiation field may reach as much as 70% in the case of lightning flashes striking a 300-m tall tower.

**Index Terms**—Electromagnetic fields, lightning to tall towers, numerical modeling.

### I. INTRODUCTION

The knowledge concerning the characteristics of electromagnetic fields generated by lightning flashes is of importance in evaluating the interaction of these electromagnetic fields with electrical networks and in the remote sensing of lightning current parameters from the measured fields. However, electromagnetic fields generated by lightning change their signature as they propagate over a finitely conducting ground due to the selective attenuation of the high frequency components (i.e. propagation effects). As a result, the peak and the peak derivatives of the electromagnetic radiation fields

measured far away from a lightning flash may deviate from their ideal values depending on the distance of propagation and the ground conductivity. Therefore, if the measured fields are used to estimate the peak current or the current derivative without any correction for propagation effects, one may obtain peak current or derivative values which could be significantly smaller than the actual values in the lightning channel.

Since the propagation effects attenuate the frequencies selectively, the way in which they modify the electromagnetic field signature depends not only on the path of propagation and the ground conductivity but also on the frequency content of the electromagnetic field. Hence, both the amount of attenuation of the initial peak and the increase in the risetime of the electromagnetic field caused by propagation effects depend on the waveshape of the originally radiated electromagnetic fields.

Most of the available studies on the propagation effects on lightning generated electromagnetic fields are confined to the case of lightning flashes striking flat ground [1]. On the other hand, studies conducted recently show that the high frequency content in the electromagnetic fields of lightning flashes striking tall structures may differ from the ones striking flat ground [2, 3]. Consequently, the way in which the initial peak and the risetime of the electromagnetic radiation field are modified by the propagation effects may differ in lightning striking tall structures in comparison to lightning striking flat ground. In this paper, we present an extension of the study presented in [3], by including structures of heights varying from 50 m to 300 m for return-stroke current risetimes varying from 0.1 to 1  $\mu$ s.

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## II. ANALYSIS OF PROPAGATION EFFECTS: THEORY AND CONSIDERED CONFIGURATIONS

### A. Propagation effects on radiation fields

In this paper, we will evaluate the propagation effects using the theory elaborated by Cooray [1], based on the early works of Norton [4], which will be summarized hereunder. The equations presented are valid for radiation (far) fields, for distances not exceeding distances of about 300 km, above which the curvature of the earth and the ionospheric effects have to be taken into account [5].

The geometry of the problem under consideration is shown in Fig. 1.

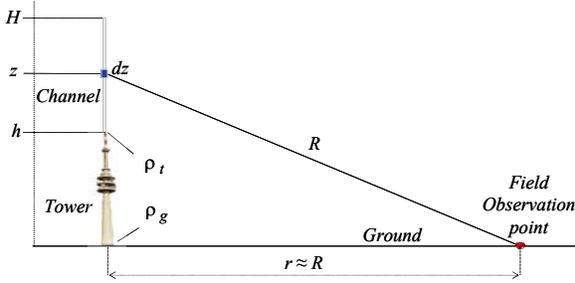


Fig. 1. Geometry relevant to the calculation of the effects of propagation over a ground of finite conductivity.

For distant observation points, assuming a perfectly conducting ground and neglecting the static and induction components of the electric field, and considering  $R \cong r$  and  $r \gg H$ , the general expression for the electric and magnetic fields for an observation point located at ground level reduces to [6]

$$E_z^{far}(r, t) = -\frac{1}{2\pi\epsilon_0 c^2 r} \int_0^H \frac{\partial i(z', t-r/c)}{\partial t} dz' \quad (1)$$

As shown by Cooray [7], the expression for the radiated vertical electric field over a finitely conducting ground is given by

$$E_{z,\sigma}^{far}(r, t) = \int_0^t E_z^{far}(r, t-\tau) S(r, \tau) d\tau \quad (2)$$

In which  $S(r, t)$  is an attenuation function which, for the case of a homogeneous ground, is given by [8, 9]

$$S(r, t) = \frac{d}{dt} \left[ 1 - \exp\left(-\frac{t^2}{4\zeta^2}\right) + 2\beta(\epsilon_r + 1) \frac{J(x)}{t} \right] \quad (3)$$

where  $J(x) = x^2(1-x^2)\exp(-x^2)$ ,  $x = t/2\zeta$

$$\beta = 1/(\mu_0 \sigma c^2) \text{ and } \zeta^2 = r/(2\mu_0 \sigma c^3).$$

$\sigma$  is the ground conductivity,  $c$  is the speed of light and  $\mu_0$  is the permittivity of vacuum.

It can be shown [7] that the third term appearing in the brackets of Eq. (3), which takes into account the effect of the displacement current in the ground, becomes negligible for propagation distances beyond 1 km, as far as the current risetime and the attenuation of the peaks are concerned. This term has been neglected in the computations presented in this paper.

### B. Model for lightning return strokes to a tall structure

We will use the so-called engineering models, extended to include the presence of a tall strike object [10]. The tower is modeled as a single, uniform and lossless transmission line. The validity of such a representation is discussed in [11].

The general equations for the spatial-temporal distribution of the current along the lightning channel and along the strike object are given by [10]

$$i(z', t) = \left[ P(z'-h) i_o \left( h, t - \frac{z'-h}{v^*} \right) - \rho_t i_o \left( h, t - \frac{z'-h}{c} \right) + (1-\rho_t)(1+\rho_t) \sum_{n=0}^{\infty} \rho_t^{n+1} \rho_g^n i_o \left( h, t - \frac{h+z'}{c} - \frac{2nh}{c} \right) \right] u \left( t - \frac{z'-h}{v} \right) \text{ along the channel} \quad (4)$$

and

$$i(z', t) = (1-\rho_t) \sum_{n=0}^{\infty} \left[ \rho_t^n \rho_g^n i_o \left( h, t - \frac{h-z'}{c} - \frac{2nh}{c} \right) + \rho_t^n \rho_g^{n+1} i_o \left( h, t - \frac{h+z'}{c} - \frac{2nh}{c} \right) \right] u \left( t - \frac{h+z'}{c} - \frac{2nh}{c} \right) \text{ along the tower} \quad (5)$$

In (4) and (5),  $h$  is the height of the tower,  $\rho_t$  and  $\rho_g$  are the top and bottom current reflection coefficients for upward and downward propagating waves, respectively,  $c$  is the speed of light,  $P(z')$  is a return stroke model-dependent function,  $u(t)$  the Heaviside unit-step function,  $v$  is the return-stroke front speed, and  $v^*$  is the current-wave speed. Expressions for  $P(z')$  and  $v^*$  for some of the most commonly used return-stroke models can be found in [12]. Throughout this study, we will use the TL model ( $P(z')=1$  and  $v^*=v$ ). Furthermore,  $i_o(t)$  is the so-called 'undisturbed current', which represents the 'ideal' current that would be measured at the tower top if the current reflection coefficients at both of its extremities were equal to zero. It is also assumed that the current reflection coefficients  $\rho_t$  and  $\rho_g$  are constant. In addition, any upward connecting leader and any reflections at the return stroke wavefront [13] are disregarded.

### C. Undisturbed current and tall tower configurations

In our computations, we consider an undisturbed current  $i_o(t)$ , given by [14]:

$$i_o(h, t) = \frac{I_{o1}}{\eta} \frac{(t/\tau_1)^2}{1+(t/\tau_1)^2} e^{-(t/\tau_2)} + I_{o2} \left( e^{-t/\tau_3} - e^{-t/\tau_4} \right) \quad (6)$$

the values of the parameters chosen are:  $I_{o1} = 9.9$  kA,  $\eta = 0.845$ ,  $\tau_1 = 0.072$   $\mu$ s,  $\tau_2 = 5.0$   $\mu$ s,  $I_{o2} = 7.5$  kA,  $\tau_3 = 100.0$   $\mu$ s,  $\tau_4 = 6.0$   $\mu$ s. This current exhibits a peak value of 11 kA, a current risetime of 0.2  $\mu$ s and a maximum time derivative of 105 kA/ $\mu$ s (see Fig. 2). Four other waveforms, shown also in Fig. 2, have also been

considered in this study. They are characterized by the same peak value (11 kA), but with different values for the risetime, namely 0.4, 0.6, 0.8 and 1  $\mu$ s.

The length of the elevated strike object has been varied from 0 to 300 m. The reflection coefficients at the extremities of the strike object were assumed to be constant and equal respectively to  $\rho_r = -0.5$  and  $\rho_g = 1$ .

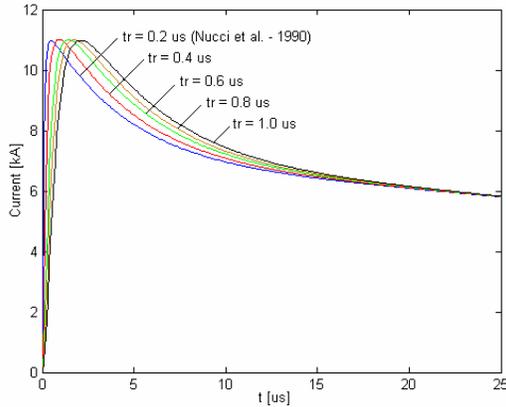


Fig. 2. Undisturbed current waveshapes adopted in this study.

### III. RESULTS AND DISCUSSION

Figure 3a shows the electric field at 100 km calculated over perfectly conducting ground for 50 m and 300 m tower heights and for the current risetime of 0.2  $\mu$ s. The radiation field corresponding to a lightning striking flat ground is also shown in the same diagram. The results corresponding to a current risetime of 1  $\mu$ s are shown in Figure 3b. Note that for small risetimes the initial peak is more or less similar for 50-m and 300-m tall towers but its width is much smaller in the case of the shorter tower. Thus one can expect the propagation effects to be more severe for the shorter tower than for the taller one. For long current risetimes both the initial peak and its width increases with increasing tower height. In this case one can expect the propagation-caused attenuation to be more severe for waveforms generated by lightning striking towers of intermediate heights.

Figure 4a depicts the radiation field generated by a return stroke having a current risetime 0.2  $\mu$ s and striking a 50-m tall tower as it propagates over a finitely conducting ground of 0.01 S/m (at 20, 50, 100 and 200 km). Note how the narrow initial peak created by the presence of the tower decreases its amplitude as the waveform propagates along a finitely conducting ground. The data corresponding to the same cases but when the conductivity is 0.001 S/m and 0.0001 S/m are shown in Figures 4b and 4c. Note how the fine structure created by the presence of the tower tends to disappear with distance.

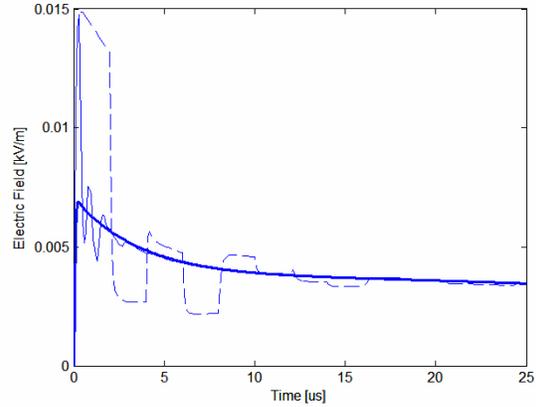


Fig. 3a. Computed electric field over perfectly conducting ground at 100 km for flat ground (solid line), a 50 m height tower (thin line) and a 300 m height tower (dashed line) for a return stroke current risetime of 0.2  $\mu$ s

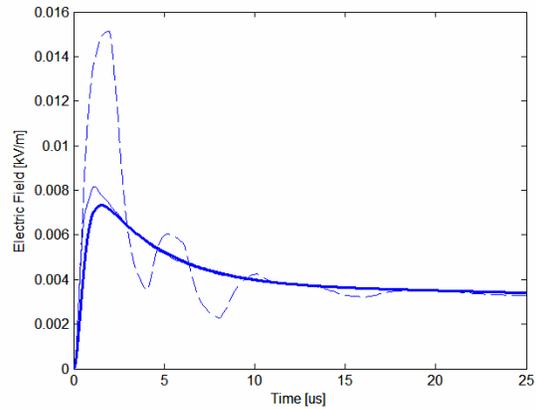


Fig. 3b. Same as Fig. 3a but for a return stroke current risetime of 1  $\mu$ s

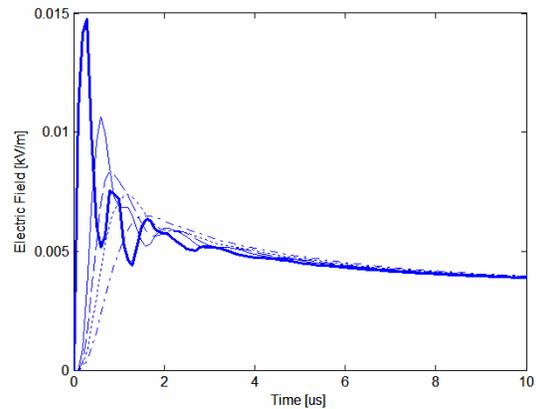


Fig. 4a. Computed radiated electric field propagating over finitely conducting ground of 0.01 S/m for a return stroke with risetime of 0.2  $\mu$ s, striking a 50m height tower. Considered distances: 20 km (thin line), 50 km (dashed line), 100 km (dotted line) and 200 km (dash-dotted line). Note that all the waveforms are normalized to 100 km assuming inverse distance dependence. This means that in the absence of propagation effects all the waveforms would have the same shape and amplitude. The thick solid line represents the 100-km field for a perfectly conducting ground.

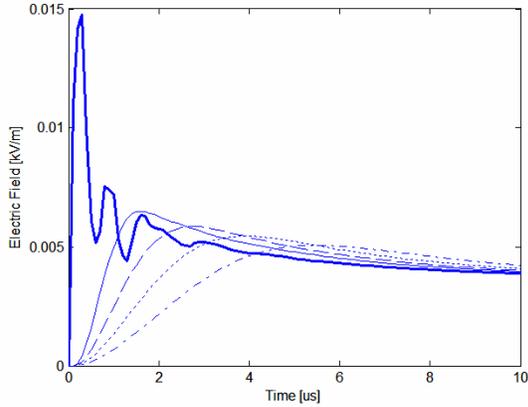


Fig. 4b. Same as Fig. 4a but for finitely conducting ground of 0.001 S/m.

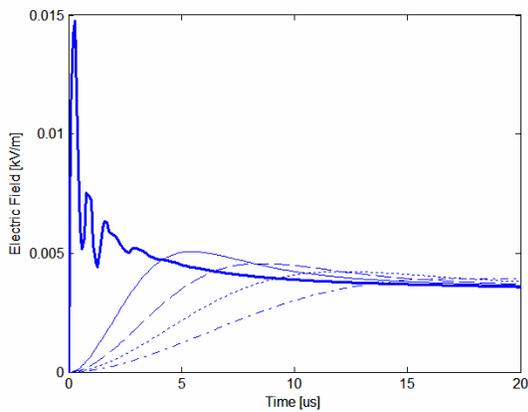


Fig. 4c. Same as Fig. 4a but for finitely conducting ground of 0.0001 S/m.

Figure 5a depicts the radiation field corresponding to 50 m tower and 300 m tower heights for the current risetimes of 0.2 after propagating 100 km over finitely conducting ground of 0.01 S/m. For comparison purposes the radiation field over perfectly conducting ground is also given in the figures. Note that the propagation attenuation of the initial peak of the radiation field corresponding to the 50 m tower is more severe. The same results for 0.001 S/m and 0.0001 S/m are shown in Figures 5b and 5c. Note that for 0.0001 S/m attenuation is more or less the same for both tower heights. Figures 6a, 6b and 6c depicts the radiation field at 100 km for the three conductivities and for the same tower heights when the risetime of the current is 1  $\mu$ s.

In order to investigate how the initial peak of the radiation field varies for different current risetimes, tower heights and ground conductivities, let us define the attenuation coefficient  $A$  as the ratio between the peak of the electric field calculated at a given distance over a finitely conducting ground to the peak of the same electric field calculated at the same distance over a perfectly conducting ground. Figures 7a to 7d show how the attenuation coefficient of a radiation field corresponding to return strokes with 0.2  $\mu$ s, 0.4  $\mu$ s, 0.6  $\mu$ s and 1.0  $\mu$ s, risetimes, respectively, varies as a function of tower

heights and conductivity. First, observe that for a given current risetime the tower height which experiences the highest attenuation slowly moves towards larger tower heights with decreasing conductivity ( Note that a large value of  $A$  means less attenuation). Second, note that for a given current risetime the minimum in the attenuation coefficient (region of highest attenuation) becomes much broader with decreasing conductivity. This effect can clearly be seen in Figures 7c and 7d. In these curves one can also notice that for a given conductivity the tower height which experiences the highest attenuation shift slowly towards large tower heights with increasing current risetime. For example, consider the cases corresponding to 0.0001 S/m. For 0.2  $\mu$ s risetime the highest attenuation is experienced by the radiation field corresponding to 50 m tower, whereas, for 1.0  $\mu$ s risetime the tower height that experiences the maximum attenuation is about 200 m. Observe also that in most of the cases the attenuation of the radiation field striking flat ground is less important than the one corresponding to return strokes striking towers.

The shape of the variation of the attenuation coefficient  $A$  as a function of the tower height can be interpreted as follows. For very short tower heights, transient processes along the tower (which contribute to the high frequency components of the radiation field spectrum) are nearly inexistent. Therefore, it is reasonable that  $A$  decreases initially with increasing tower heights. On the other hand, the transient processes (multiple reflections) for very tall structures tend also to affect less the initial rising portion of the field because the first reflection from the ground affects the field waveform well after it has reached his peak. This explains the increase of the attenuation coefficient for taller towers. In between, when the current risetime is similar to the propagation time along the tower (0.2  $\mu$ s corresponds to a 60-m tall tower, 0.4  $\mu$ s to 120 m, and so on), the initial rising portion of the field is most affected by the multiple reflections, resulting the most significant attenuation.

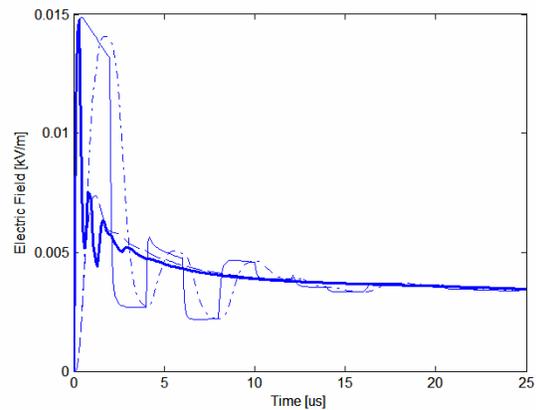


Fig. 5a. Computed radiation electric field over finitely conducting ground of 0.01 S/m at 100 km produced by a return stroke with risetime of 0.2 $\mu$ s striking a 50m height tower (dashed line) and a 300m height tower (dash-dot line). The radiation fields over perfectly conducting ground at the same distance are also included (thick solid line: 50 m; thin solid line: 300 m).

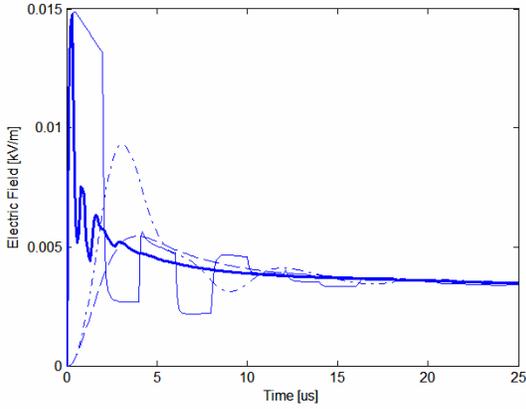


Fig. 5b. Same as Fig. 5a but for finitely conducting ground of 0.001 S/m.

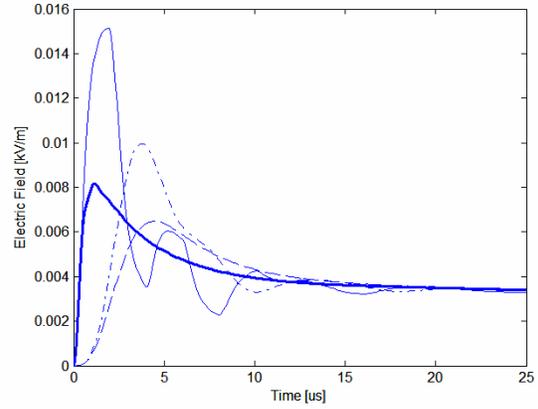


Fig. 6b. Same as Fig. 6a but for finitely conducting ground of 0.001 S/m.

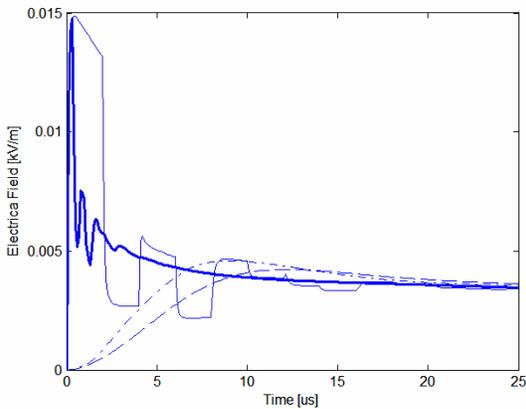


Fig. 5c. Same as Fig. 5a but for finitely conducting ground of 0.0001 S/m.

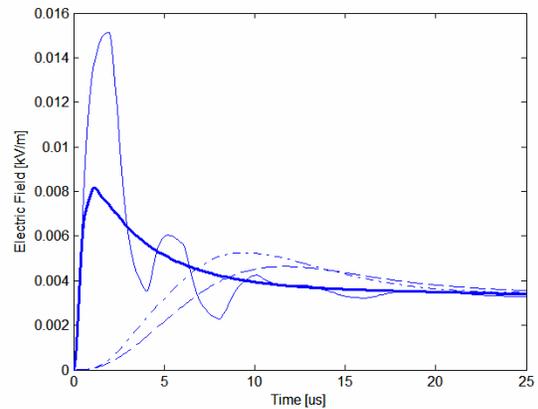


Fig. 6c. Same as Fig. 6a but for finitely conducting ground of 0.0001 S/m.

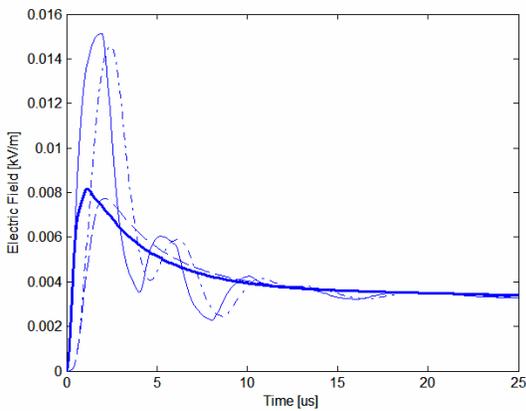


Fig. 6a. Computed radiation electric field over finitely conducting ground of 0.01 S/m at 100 km produced by a return stroke with risetime of  $1\mu\text{s}$  striking a 50m height tower (dashed line) and a 300m height tower (dash-dot line). The radiation fields over perfectly conducting ground at the same distance are also included (thick solid line: 50 m; thin solid line: 300 m).

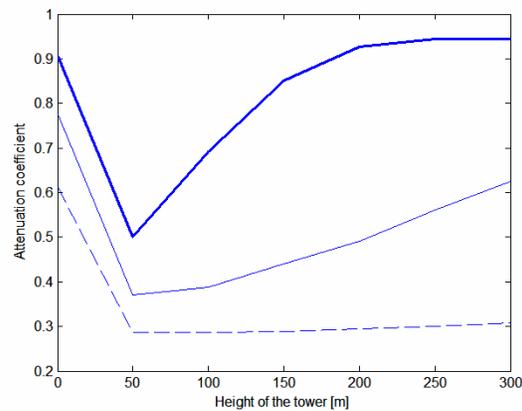


Fig. 7a. Attenuation coefficients of the radiation fields at 100km over finitely conducting ground of 0.01 S/m (solid line), 0.001 S/m (thin line) and 0.0001 S/m (dashed line) produced by a return stroke current with  $0.2\mu\text{s}$  risetime as function of the height of the tower.

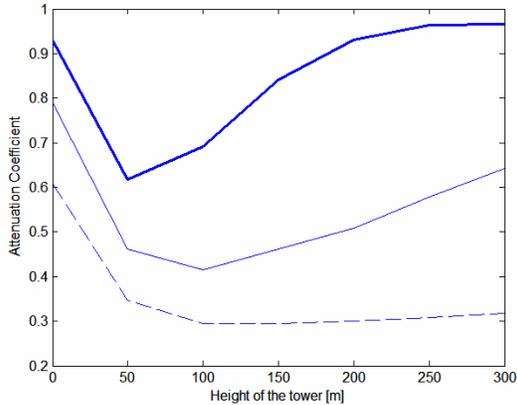


Fig. 7b. Same as Fig. 7a but with a return stroke current risetime of  $0.4\mu\text{s}$ .

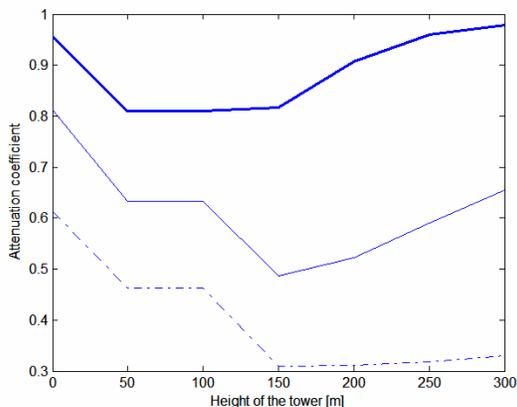


Fig. 7c. Same as Fig. 7a but with a return stroke current risetime of  $0.6\mu\text{s}$ .

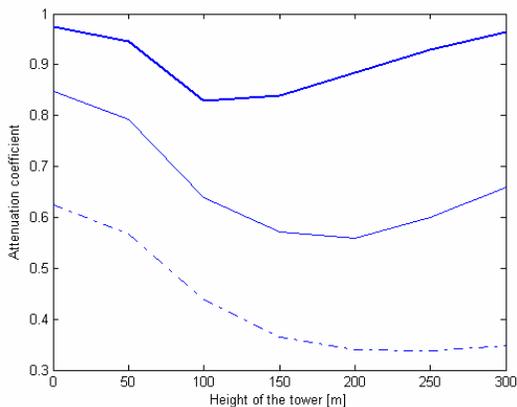


Fig. 7d. Same as Fig. 7a but with a return stroke current risetime of  $1\mu\text{s}$ .

#### IV. CONCLUSIONS

The results presented in this paper show how the finitely conducting ground modifies the signature of the radiation field of return stroke striking tall towers. The results are presented for different tower heights and for different ground conductivities varying the current risetime in the return stroke model. The results show that the attenuation of the initial peak of the radiation field resulting from the propagation over finitely conducting ground depend

strongly on the risetime of the current, the tower height and the ground conductivity. In general, the attenuation of the radiation field of lightning flashes striking towers is larger than that striking flat ground. In the case where the ground conductivity is  $0.0001\text{ S/m}$  the attenuation of the peak radiation field may reach as much as 70% in the case of lightning flashes striking a 300 m tower.

It was found that the transient processes along the tower result in a more significant attenuation of the radiated field when the current risetime is similar to the propagation time along the tower.

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