# Site Error Correction with Time Information 

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#### Abstract

Site error correction is an important process in the operation of a lightning location system based on magnetic direction finder technology. Different algorithms have been published in literature to correct systematic angle errors [Schulz and Diendorfer, 1993; Soerenson, 1995]. These algorithms use the angle information only for the correction, because the old version of the LLP system only provided the angle information of a flash.


With the new IMPACT technology, in addition to the angle also the arrival time of the signal at the site is measured. So it is possible to correct the site errors more efficiently by including time information in the site error algorithms.

In this paper we show, how site error correction can be performed including the time information and how the use of the additional time information improves the results.

In addition to the correction of the angular error it is also possible to correct the propagation error of the signal approximately. The improvement in data quality of the localized strokes due to the application of a time correction is evaluated.

## 1. INTRODUCTION

For the operation of a magnetic direction finder system detailed knowledge of the systematic direction errors (site errors) is very important. From literature several investigations about this topic are available [Horner 1954; Ito and Goto, 1957]. In 1994 LLP has introduced a new type of direction finder, the IMPACT sensor. The main improvement to the previous version is the time synchronization of the sensor clock by GPS signals providing an absolute time accuracy of better than 300 ns . The new sensor determines the angle of incidence and arrival time of the field radiated by the lightning stroke. Even in the operation of the new IMPACT technology the correction for the site error is still important. For the identification of site errors there are different methods available from literature, e.g. optical measurements [Mach et al., 1986] or a reciprocal method [Schütte, 1987]. We are using a statistical approach because with this, the least effort is
necessary. For this method a certain number of strokes, registered by at least three direction finders, is required. This allows calculation of the stroke position by applying a least square method (optimized location). Several papers in the lightning literature [Hiscox et al., 1984; Passi and Lopez, 1989; Lopez and Passi, 1991] explain the basical concept of site error correction.

## 2. THEORY OF SITE ERROR CORRECTION

The statistical site error correction applied in this paper is an iterative process with basically three steps.

1) Calculation of an optimized position with the aid of a least square algorithm
2) Calculation of the angle deviations between measured angles and angles to the optimized locations from step 1
3) Correction of the measured angles with the angle deviations and restart with step 1.
We are using a parametric representation for the site error of the following form

$$
\begin{equation*}
\beta_{i}\left(\theta_{i}+R O_{i}\right)=\sum_{j=1}^{N_{h}}\left[a_{i j} * \sin \left(j\left(\theta_{i}+R O_{i}\right)\right)+b_{i j} * \cos \left(j\left(\theta_{i}+R O_{i}\right)\right)\right] \tag{1}
\end{equation*}
$$

where $\Theta$ is the measured angle, RO is the rotation of the direction finder, $a$ and $b$ are the site error parameters, $\mathrm{N}_{\mathrm{h}}$ is the number of harmonics and $i$ is the direction finder number. This kind of representation is applied due to the reason that the Advanced Position Analyzer (APA) of the LLP detection system uses the same function to correct site errors. Therefore the true angle a from the sensor to the flash location is calculated from

$$
\begin{equation*}
\alpha_{i k}=\theta_{i k}+R O_{i}+\beta_{i}\left(\theta_{i k}+R O_{i}\right)+e_{i k} \tag{z}
\end{equation*}
$$

where $\Theta_{\mathrm{ik}}$ is the measured angle from the ith direction finder to the kth flash. The rotation RO of a direction finder is part of the site error. If for some reason the antenna is rotated by a known angle all the site error parameters will not change except the rotation RO. This is an
advantage of this type of representation. Although in literature the site error representation is often used in form of a twocycle sinusoidal function [Passi and Lopez, 1989; Ito and Goto, 1957], we are also including the odd harmonics for the following reasons:

1) there is some evidence for the existence of odd harmonics in site errors [Kawamura et al., 1988]
2) calculation time is not increasing, if two or more harmonics are calculated
3) site error analysis revealed a significant third harmonic at one site in Austria.

## Step 1: Optimization of the location

Due to the implementation of the GPS time synchronization in the new IMPACT direction finder, it is also possible to use the time information for the calculation of the stroke location. It was our intention to make the calculation as accurate as possible and therefore we are using an elliptic earth model for the angle calculation and for the calculation of the arc length, although the manufacturer of the MDF System uses a spherical earth model for the angle calculation. The so called WGS84 ellipsoid is applied for all calculations [Hofmann-Wellenhof et al., 1992]. The azimuth of a direct normal section $a_{12}$ (see fig. 1) is used for the angle and the arc length of the normal section is used for the arc length [Torge W., 1991]. A normal section is the intersection between the ellipsoid and a vertical plane defined by the surface normal at point $P$. For lightning location the normal section is defined by the surface normal of the point of view $\mathrm{P}_{1}$ (DF site) and the point of interest $P_{2}$ (striking point).


Fig. 1: Direct ( $\alpha_{12}$ ) and inverse ( $\alpha_{12}{ }^{*}$ ) normal sections for point $P_{1}$

The azimuth $\alpha$ is a function of the latitude $\varphi$ and the longitude $\lambda$ of the point of interest

$$
\begin{equation*}
\alpha=\alpha(\varphi, \lambda) \tag{3}
\end{equation*}
$$

and the arrival time

$$
\begin{equation*}
t_{a}=t_{i}+\frac{D}{v} \tag{4}
\end{equation*}
$$

| where | $t_{a}$ | $\ldots$ | arrival time |
| :---: | :---: | :---: | :--- |
|  | $D$ | $\ldots$ | distance <br> lightning |
|  |  |  |  |
|  | $t_{i}$ | $\ldots$ | impact time |
|  | V | $\ldots$ | propagation velocity |

The arrival time is a function of the impact time and the distance $D$ to the striking point. Because the azimuth a does not depend on the impact time an optimization of the impact time with angle information only is not possible.

With the algorithm presented in this paper we are able to optimize the location on the ellipse using the

1) angle information only (at least 3 direction finders reporting the stroke are required)
2) time information only (at least 4 direction finders reporting the stroke are required)
3) angle and time information (at least 2 direction finders reporting the stroke are required).

Because expressions for angle and distance on an ellipse are nonlinear in latitude and longitude of the striking point, the optimization has to be done iteratively with a least square algorithm.

## Step 2: Calculation of the systematic angle deviations

According to equ. (1), where the site error is a function of the sum of the measured angle $\Theta$ and the rotation RO, this function is nonlinear in the parameter RO and therefore has also to be solved iteratively with a least square algorithm. The result is a first approximation for the coefficients $\mathrm{a}_{\mathrm{ij}}$ and $\mathrm{b}_{\mathrm{ij}}$ of equation (1).

Step 3: Correction of the measured angles with the systematic angle deviations and restart with step 1
After determination of this first approximation of
the site error parameters, it is necessary to correct the measured angles with the determined error values and to repeat this procedure until the algorithm has reached an optimum solution. We define an optimum solution as a solution, where the sum of all reduced Chi-square values does not change significantly by the last step of the iteration. The Chi-square value is a measure for describing the disagreement between measured values and optimized values. When Chi-square is normalized by the degrees of freedom in the optimization procedure, this value is referred to as the reduced Chi-square. By the fact that the reduced Chi-square changes its values depending on the used location algorithm (the number of degrees of freedom is different even with the same number of involved direction finders), the permissible change of the sum of the Chi-square is also different for the distinct algorithms.

## 3. COMPARISON OF SITE ERROR CORRECTION WITH DIFFERENT LOCATION METHODS

For this comparison we used lightning data from the Austrian lightning detection network ALDIS [Diendorfer et al., 1992; Diendorfer et al., 1994]. The data have been selected from our database in order to represent a set of almost regularly distributed data over the whole area covered by the network.

For the site error correction with angle information only (referred as method (A)) optimized solutions are required. Therefore an over determined equation system has to be solved. Otherwise we cannot determine an angle difference between the measured angle $\Theta$ and the angle $\alpha$ to the optimized location.

We have compared results of site error corrections based on all three fundamental methods of location determination (angle only (A), time only (T), angle and time (A\&T)). As a first remarkable result the site error correction including time information for the location determination converges much faster than the others. This can be seen by the comparison of the "canonical Chi-square". The "canonical Chi-square" is the sum of the Chi-square values of all strokes involved in the site error correction. Fig. 2 shows that after some iterations there is no more improvement of the canonical Chi-square value. Site error
correction with time information only converges within one iteration and is therefore not shown in fig. 2. A further iteration is useless, because the determined site error parameters do not effect the location optimization.


Fig. 2: Canonical Chi-square as a function of iterations

Fig. 3 shows a comparison of the standard deviation of the residual site errors for each individual direction finder. It is obvious that a site error correction with angle information only (A) provides better results. Using angle and time information (A\&T), the algorithm searches for a minimum of the deviations of both information, and this is not necessarily the optimum solution for angle information only.


Fig. 3: Standard deviation of the residual site error

Although this is not really a way to estimate the location accuracy, it is possible to evaluate the improvement due to the site error correction by
a comparison of the Chi-square distributions before and after the site error correction. For testing the improvement due to the site error correction, it is necessary to use a data set different to the data set used for the determination of the site error parameter. Again, we are using a set of lightning locations distributed almost regularly over the whole network. Fig. 4 shows a comparison of the Chisquare distributions without and with site error correction. The distributions are calculated with angle and time information (A\&T). The standard deviation of the angle information was assumed to $\sigma_{\mathrm{a}}=1^{\circ}$ and the standard deviation of the time information to $\sigma_{\mathrm{t}}=1 \mu \mathrm{~s}$. The applied site error parameters have been derived with angle and time information (A\&T).


Fig. 4: Chi-square distribution before and after site error correction

Of course not all strokes have a reduced Chisquare after site error correction. In the distribution shown in fig. 3 about $86 \%$ of all analyzed strokes have a smaller Chi-square value after site error correction than before site error correction. Soerensen [1995] realized, that site error correction with angle only ( $\mathbf{A}$ ) is not possible outside the network. In our case of site error correction with angle and time information (A\&T), the remaining 14\% of strokes with higher Chi-square values after the site error correction have locations outside the network. This could be an indication that also with time information site error correction outside the network does not make sense.

In a next step we compared Chi-square distributions for site error parameters derived
from angle information (A) and from angle and time information (A\&T). It is remarkable that the results are always slightly better for the site error parameters derived from angle and time information (A\&T) in opposite to the smaller residual site errors calculated by using angle information only (A).

This leads us to the conclusion that it is preferable to do the location optimization in a site error algorithm with time information, additionally to the angle information. There is not a significant difference between the site error correction from angle and time information (A\&T) or time information only ( $\mathbf{T}$ ).

## 4. TIME ERROR CORRECTION

After the correction of the systematic angle error we analyzed more carefully possible time errors of the signals. The manufacturer of the direction finding system states an absolute time error of the GPS clock of 300 nanoseconds. For the location algorithm of course not the error of the absolute time of the signal is important but the time error of the time differences. We expected that this time error should be much smaller than 300 nanoseconds. Therefore we assume that the relative time error is limited by the resolution of the DF time being 119 nanoseconds.

The stroke time used by the Position Analyzer for the calculation of the location is neither the time when the signal exceeds the threshold nor the time of the signal peak, but the time of the signal onset. This time is calculated from the time when the signal exceeds the threshold, the time of the peak, the threshold value and the peak value of the signal. With this information, it is possible to calculate the signal onset. The reason is to reduce timing differences due to waveform distortion caused by the finite ground conductivity [Uman et al., 1976].

In a first step we looked at the time differences $\Delta t$ (time error) between optimized arrival time and measured arrival time as a function of distance for a certain direction finder (fig. 5). Equ. (5) shows the calculation of the time differences,

$$
\begin{equation*}
\Delta t=\left(t_{i c}+\frac{D}{v}\right)-t_{a m} \tag{5}
\end{equation*}
$$

where $t_{a m}$ is the measured arrival time, $t_{i c}$ is the optimized impact time, D is the distance to the stroke and $v$ is the propagation velocity of the signal. All used locations are optimized with time information only ( $\mathbf{T}$ ) and no special direction for the DF number 4 is considered. We determined a standard deviation for the time errors of about $1.4 \mu \mathrm{~s}$. If there is no systematic time error, it will be no correlation between distance and time error.


Fig. 5: Time differences for DF number 4
Fig. 5 shows a significant correlation between these two values ( $r=0.67$ ). In a next step we tested, how different values of the arc length influence these results. We determined a minimum value of the canonical Chi-square for arc lengths $0.07 \%$ longer than the arc length on the ellipsoid. Fig. 6 shows the time deviations calculated with this increased arc length.


Fig. 6: Time differences calculated with increased arc length

The result of using arc lengths stretched by $0.07 \%$ is a reduced standard deviation of $0.6 \mu \mathrm{~s}$ and in this case there is almost no correlation between the time difference and the distance $(r=0.22)$. For a data set of about 800 strokes optimized with the increased arc length, the canonical Chi-square reduces from 3470 to 1010.

## 5. DISCUSSION

As we could show, the time information for the site error correction has a major effect on the convergence of the site error algorithm. The time information also influences the result of the correction. Better Chi-square values are achieved, when time information is included in the site error algorithm.

We found that also the time information has a systematic error related to the calculated propagation distance.

One or several of the following reasons could cause this time error:

1) The real propagation velocity over finite ground conductivity is a little bit lower than the speed of light.
2) The calculations are done without any consideration of altitudes on the ellipsoid (DF site and striking point are assumed at the surface of the ellipsoid).
3) Due to different obstacles (mountains) along the propagation path from the stroke location to the direction finder, the real propagation path does not follow exactly an ellipse, and the arc length is increased.

The first and the second reason should be of second order, but the third could be a reason for time errors, when arc lengths equal to the arc lengths on the ellipse are used.

It would be interesting if similar results were achieved in areas that are flatter than most of the regions in Austria. It is also worth to note, that this time error caused by decreased arc length is different to the so called "Alps effect" reported for the time of arrival location system in Switzerland [Montandon, 1992]. The Alps effect is caused by a time delay of the peak due to finite ground conductivity. The effect reported in this paper is independent of ground conductivity and a result of the geometry of the propagation path.

As an additional result for the location

